

日程安排

ABSTRACT. Let $\{a_1, a_2, \dots, a_n, \dots\}$ be a sequence of complex numbers which has at most polynomial growth and satisfies an extra assumption. In this talk, inspired by a recent work of Sasane, we give an explanation of the sum

$$a_1 + 2a_2 + 3a_3 + \dots + na_n + \dots,$$

and more generally, for any $k \in \mathbb{N}$, the sum

$$1^k a_1 + 2^k a_2 + 3^k a_3 + \dots + n^k a_n + \dots,$$

from the viewpoint of distributions. As applications, we explain the following summation formulas

$$1^k - 2^k + \dots + (-1)^k = E_k(0)$$

$$\sum_{n=1}^{\infty} \frac{1}{n^k} = \frac{1}{k-1} \sum_{n=1}^{\infty} \frac{1}{n^{k-1}}$$

where $E_k(x)$ is the Eulerian polynomial. The other summation formula is based on the well-known formula $\sum_{n=1}^{\infty} \frac{1}{n^k} = \frac{1}{k-1} \sum_{n=1}^{\infty} \frac{1}{n^{k-1}}$ and the fact that $\sum_{n=1}^{\infty} \frac{1}{n^k} = \frac{1}{k-1} \sum_{n=1}^{\infty} \frac{1}{n^{k-1}}$ holds for $k > 1$. The other summation formula is based on the fact that $\sum_{n=1}^{\infty} \frac{1}{n^k} = \frac{1}{k-1} \sum_{n=1}^{\infty} \frac{1}{n^{k-1}}$ holds for $k > 1$.
